



# The shape of utility functions and voter attitudes towards risk

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## ABSTRACT

The key concern of this contribution is how voters transform distances within a political space into voter utility, i.e. whether theoretical and/or statistical models of vote choice should employ linear or quadratic loss functions to adequately capture spatial utility: First, the selection of an appropriate distance metric is a key concern in the specification of theoretical and statistical models of vote choice. Secondly, these options transcend the domains of mere technical modeling choices, but address attitudes towards risk, and directly relate to voter reactions to widespread phenomena such as party ambiguity and/or voter uncertainty.

The empirical analysis rests upon the rich data provided by the collaborative project “The Comparative Study of Electoral Systems” (CSES). Comparative data analyses across heterogeneous national and electoral contexts clearly demonstrate that voters are, on average, much less risk-averse than assumed by the vast majority of empirical and theoretical contributions. Instead, we find that voters in mass elections are by and large neutral towards risk and are not systematically repelled by party ambiguity and (candidate-induced or perceptual) voter uncertainty.

## 1. Introduction

The spatial theory of voting posits that voters and parties may be located within a political spaces and that the ideological or programmatic distance among the various alternatives determines electoral utility and vote choice (Downs, 1957; Enelow and Hinich, 1984). This article explores the conversion of spatial distances among voters and electoral platforms into voter utility. The imminent literature has in almost any case, and usually without any specific discussion or justification, assumed representations of utility loss that also imply diverging voter attitudes towards risk and uncertainty. Instead of *a priori assuming* any specific shape of utility loss functions and risk attitudes, this contribution empirically *estimates* both issue salience and attitudes towards risk across a wide range of real-world elections. Therefore, we arrive at both more adequate and realistic models of spatial voting and obtain empirically backed insight into the electoral consequences of voter uncertainty.

The identification of a realistic distance metric comes with more significant implications for electoral politics and research when party ambiguity and/or voter uncertainty are considered. Both theoretical and empirical analyses of vote choice need to consider the shape of utility profiles and voter attitudes towards risk so as to guard against the misspecification of key modeling choices. When models comprise of two or more dimensions, the shape of utility curves may modify the rank order of the candidates or parties competing. Provided that no candidate or party may be located in a political space with absolute certitude, but rather with varying degrees of uncertainty, valid assessments of risk attitudes are essential for any realistic model of vote choice.

These aspects transcend the domain of mere technical modeling choices, but are of direct substantial significance. Consider a stylized example when a certain party  $p$  and an uncertain alternative  $p'$  compete and are located with the same distance from a voter ideal point  $v$ . Since in both cases the distance of voter ideal points and party positions is identical, both alternatives differ only in the certainty/uncertainty of their spatial positions which may be captured and illustrated by a statistical distribution that is centered at the respective “true” position. In this case, a rational voter with a risk-averse, concave utility profile will prefer the certain platform  $p$  to the uncertain alternative  $p'$ , a risk-neutral voter, characterized by a linear loss function will be indifferent among  $p$  and  $p'$ , and a risk-acceptant voter with an at least partially convex utility function will be ready to gamble with her vote and select the uncertain  $p'$  (cf., for instance, Alvarez, 1998; Enelow and Hinich, 1984; Shepsle, 1972).

To date, there is no settled consensus on how voters convert spatial distance into electoral utility. The formal and empirical modeling literature in the wake of Davis et al. (1970) and Enelow and Hinich (1984) assumed that voter utility tends to recede at an increasing rate when candidates or parties move further away from the voter ideal point and formalized this idea by the provision of quadratic utility loss. These models also imply that voters are supposed to be repelled by candidates or parties that adopt ambiguous political platforms (Shepsle, 1972) or may merely be located with substantial uncertainty (Alvarez, 1998; Enelow and Hinich, 1981; Bartels, 1986).

However, some more recent contributions challenge this standard assumption of the “Neo-Downsian” modeling tradition and suggest that electoral utility declines at a constant rate when distances among voter and candidate or party positions increase (Degan and Merlo, 2009;

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Eguia, 2012; Grynaviski and Corrigan, 2006; Humphreys and Laver, 2010; Patty et al., 2009). Formally, this is embodied by the application of linear instead of quadratic loss functions. Linear loss profiles imply that voters are, on average, indifferent to risk and neither repelled nor attracted by party ambiguity and voter uncertainty (Berinsky and Lewis, 2007; Grynaviski and Corrigan, 2006).

In this article, our take is that the choice among both distance metrics is essentially an empirical issue, and potential choices need to be backed up and supported by solid and strong empirical evidence. We therefore strive to identify the empirical shape of voter utility functions across a wide range of heterogeneous polities and elections. Building upon a systematic-comparative framework, we focus on a set of ninety post-election surveys conducted from 1996 to 2011 in the wake of elections to national parliaments, which have been compiled from the first three waves of the “Comparative Study of Electoral Systems” (CSES; <http://cses.org/>).

We present our argument in a number of consecutive steps: In the next section, we briefly discuss theoretical issues of spatial model specification, derive and present the key quantities of interest, i.e. parameters that specify issue salience and attitudes towards risk (section 2). Subsequently, we turn from theoretical to empirical foundations, present the CSES datasets, and operationalize core variables that capture spatial and non-spatial utility components (section 3). Based on these building blocks, the following section presents our empirical findings. Both for simple baseline models with fixed party positions and with more sophisticated models that consider and model voter uncertainty, we find that voters generally gravitate towards risk-neutral attitudes and never behave in the strictly risk-averse fashion assumed by the “Neo-Downsian” standard. Evidently, linear loss functions provide a significantly better representation of voter calculus than standard squared loss functions (section 4). We also show that patterns of risk aversion or risk-acceptance are stable for successive elections within the country contexts. The final section briefly wraps up the core findings and lines out some avenues for future research (section 5).

## 2. A priori assumptions and theoretical specifications

This section begins by briefly restating key elements of the spatial theory of voting. Both their deterministic and probabilistic specifications agree that a voter  $v$  selects the candidate or party  $p$ , which provides her with the highest expected utility  $u(v, p)$ . The “policy” or “spatial” part of voter utility is defined by a loss function, which implies that voter utility declines with increasing distance among voter ideal points and party positions in a one-dimensional political space:

$$u(v, p) = -L_{\beta}(v, p) = -|v_i - p_{i,j}|^{\beta}$$

Common model specifications usually evaluate spatial proximity either by squared Euclidean or by linear distances, and both squared and linear distances are special cases of this more general mathematical expression: Common quadratic loss functions (à priori set to  $\beta = 2$ ) imply concave voter utility functions; marginal utility loss increases when candidates or parties move further away from the voter's ideal point. Euclidean utility functions assume voter aversion to electoral risk and continue to be the standard approach within the “Neo-Downsian” modeling tradition (cf., among many others, Adams et al., 2005; Davis et al., 1970; Enelow and Hinich, 1984; Feddersen, 1992; Merrill and Grofman, 1999; Schofield and Sened, 2006).

In contrast, linear loss functions (à priori set to  $\beta = 1$ ) posit that voter utility recedes at a constant rate with increasing spatial distance among voters and parties. This distance metric, which assumes grosso modo risk-neutral voters, traces back to the original specifications by Hotelling (1929) and Downs (1957), and it continues to be maintained by a strong minority of contributions to the current literature on spatial modeling (cf., for instance, Degan and Merlo, 2009; Eguia, 2012; Grynaviski and Corrigan, 2006; Humphreys and Laver, 2010; Kramer,

1977; Patty et al., 2009; Wittman, 1973, 1977).

To à priori assume one or the other distance metric is from our perspective deeply problematic. In contrast, to empirically estimate the shape of voter utility functions is a complex endeavor, but comes with some significant advantages. First and foremost, estimates of  $\beta$  are backed and supported by actual empirical evidence. Moreover, empirically derived shape parameters are not tied to only two alternative values, i.e.  $\beta = 1$  or  $\beta = 2$ , but allow for more realistic and fine-grained evaluations of voter attitudes towards risk. In principle, meaningful values of  $\beta$  may be taken from the domain of positive real numbers. Note that  $\beta = 0$  is not a reasonable value, because  $d^{\beta} = d^0 = 1 \forall d \neq 0$ , and negative values of  $\beta$  also prevent any substantively meaningful interpretation, because that would substantively imply that the scales are reversed and voter utility *increases* with the distance of electoral platforms from voter ideal points.

Against this background, we briefly present five potential scenarios for different values and ranges of  $\beta$  along with their substantive implications:

$\beta > 2$  Shape parameters larger than two refer to a domain of *extreme risk aversion* when voters do not react much to smaller deviations from their ideal points, but utility diminishes very abruptly when spatial distances to political parties get larger.

$\beta \sim 2$  Estimated shape parameters of (about) two still assume substantive levels of *risk-aversion* which correspond to the assumptions of the “Neo-Downsian” standard model.

$1 < \beta < 2$  When values of  $\beta$  range between one and two, voters are supposed to be *vaguely risk-averse*, but with decreasing values of  $\beta$  risk aversion also decreases.

$\beta \sim 1$  With shape parameters of (about) one, voter utility diminishes at a constant rate and voters are supposed to be *indifferent* towards risk.

$0 < \beta < 1$  Shape parameters which are larger than zero and smaller indicate that voter utility decreases at a decreasing rate when spatial distances among voters and parties increase. This also implies that voters tend to be *risk-acceptant*. Whenever voters are dissatisfied, believe their vote has little impact on political decisions, or are generally willing to gamble with their ballot, this may be a realistic scenario. Nonetheless, the formal and empirical literature have usually not systematically incorporated risk-acceptant preferences.

In the subsequent step, we translate the theoretical into a statistical model that closely matches the key arguments laid out above. We employ stochastic discrete choice models so as to capture the salience of spatial and non-spatial determinants of vote choice, to assess the shape of average voter utility functions, and to provide measures of uncertainty for descriptive and causal inferences (for a general discussion cf. Train, 2009; for more specific applications to vote choice Alvarez and Nagler, 1998 and Dow and Endersby, 2004).

In conditional logit models of electoral choice, the probability that a voter  $i$  selects party  $j$  is given by:

$$\Pr(v_i = j) = \frac{\exp[u(v, p)]}{\sum_{j=1}^J \exp[u(v, p)]}$$

Our statistical random utility model is operationalized by the key variables of the spatial model and parameterized by a set of coefficients which capture the salience of spatial and non-spatial utility components, the shape of utility curves, and the probabilistic disturbance term. General utility of party  $j$  to voter  $i$  is provided by:

$$u(v, p) = -\alpha|v_i - p_{i,j}|^{\beta} + \theta_j D_i + \lambda_j$$

This utility function is composed of a spatial part which addresses individual- and party-specific geometric distances among voters and parties in an  $n$ -dimensional political space ( $|v_i - p_{i,j}|$ ). Its non-spatial components include a battery of independent variables which capture non-spatial, individual-specific determinants of vote choice such as party identification, satisfaction with democracy, political sophistication, education, age, and gender ( $D_i$ ). Finally, general voter utility

includes a vector of party-specific constants which capture unmodeled (alleged “valence”) features of political parties. Note that we only include these party-specific constants to guard against model misspecification, while we are not ready to assign these coefficients a substantive meaning such as candidate or party quality, competence, or “valence” (as suggested, among many others, by Schofield and Sened, 2006).

Some additional model parameters are estimated from the data:  $\alpha$  indicates the salience of spatial considerations within the general utility function and is expected to be positive and substantively significant.  $\beta$  characterizes the shape of the utility functions and thus indicates voter attitudes towards risk. The parameter vector  $\lambda_j$  captures the salience of unmodeled effects at the party level, and the coefficient vector  $\theta_j$  addresses the effects of individual-specific predictors of vote choice.

### 3. Comparative election studies, data, and variables

This contribution explores a theoretically sound and empirically adequate choice for the distance metric, i.e. for the shape of utility curves, in spatial models of voting. To date, the parallel election surveys gathered by the CSES project are *the* most unique and exhaustive data source for comparative electoral studies. Our evaluation of electoral salience and voter attitudes towards risk builds upon a cumulation of the first three waves of CSES, which covers a heterogeneous set of 129 election survey segments compiled from more than forty national contexts. The “CSES Module 1–3 Harmonized Trend File” has been compiled and edited at the Berlin Social Science Center (WZB) by Heiko Giebler, Josephine Lichteblau, Antonia May, Reinhold Melcher, Aiko Wagner, and Bernhard Weßels. (Data and documentation are available via the main CSES website <http://cses.org/datacenter/trendfile/trendfile.htm>.)

The cumulative dataset covers an overall time span from 1996 to 2011. We have decided to exclusively focus on elections to national parliaments (to lower houses in bicameral systems), therefore the dataset compiled for our analyses includes ninety post-election surveys from forty-four different, heterogeneous polities. Most of the election segments are based upon interviews with roughly 1000 to 2000 eligible voters with a minimum of  $N = 860$  voters in Great Britain (2005) and a maximum of  $N = 4495$  voters in Canada (1996). We believe that this rich database provides a unique opportunity to generalize current findings on the conversion of spatial distances among voters and parties into electoral utility and on the shape of utility curves.

The comparative datasets provided by the CSES project cover survey items and additional empirical data on all items required to specify unified models of spatial and non-spatial utility components. The post-election survey segments include a common module that gather information on reported vote choice, individual-specific voter self-placements ( $v_i \in [0,10]$ ) and idiosyncratic, alternative-specific party placements on the left-right dimension ( $p_{i,j} \in [0,10]$ ). In addition, the common survey modules fielded by the CSES project also cover non-policy controls such as party identification, satisfaction with democracy, a battery of items that capture political knowledge, and key demographic features of the voters such as age, education, and gender. (For additional details concerning the original question wording and the operationalization of the respective variables please cf. Appendix B.)

However, the selection of the CSES data also comes with some restrictions. While we value the rich data on voter and party positions on a common left-right scale, the CSES questionnaires do not collect similar data for other issue dimensions, and thus the selection of this data source also implies the restriction of our scope to one ideological dimension instead of multiple policy scales and to uni-instead of multi-dimensional political spaces. However, with the partial exemption of the European Election Studies, we are not aware of any survey material which includes both voter and party positions on a number of issue dimensions and in an extensive, comparative format.

### 4. The shape of utility curves and attitudes towards risk

This article is concerned with voter attitudes towards risk and uncertainty. Our two core perspectives also structure the setup and presentation of the empirical evidence. Initially, we derive findings on the risk-orientations of the diverse electorates stacked in the CSES trend file by specifying baseline discrete choice models of vote choice under certainty. Subsequently, we extend these specifications to also address substantive issues of vote choice under risk and uncertainty. We present and discuss comprehensive, fully specified models of vote choice when party positions are blurred and/or perceived with varying levels of uncertainty.

#### 4.1. Baseline models of vote choice under certainty

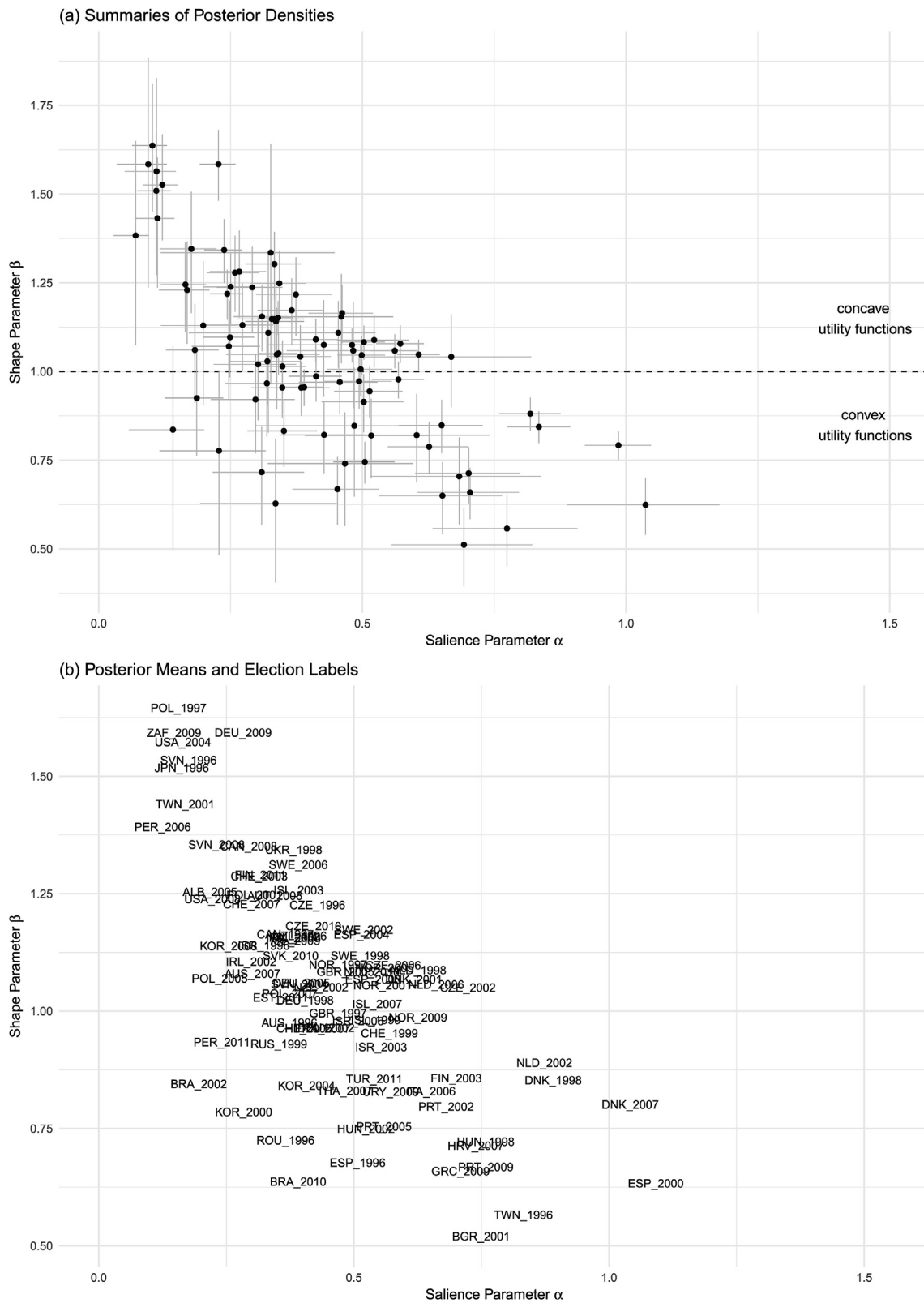
We begin the presentation of empirical results with simple models of elections to national parliaments. The data is taken from the cumulated CSES dataset and covers ninety individual elections (and thus ninety post-election survey segments) to national parliaments from 1996 to 2011. Statistical models of vote choice are assessed separately for each individual election context. The key predictor variables are spatial distances among voters and parties so that all actors, and the key quantities of interest to be estimated from the data are the salience of the ideological left-right conflict ( $\alpha$ ) and the shape of the utility function in the electorate ( $\beta$ ).

The basic model at hand may be conveniently assessed either by maximum likelihood or by Bayesian MCMC. For simple models of electoral choice, these alternative techniques provide generally identical results. However, advancing towards more complex specifications, we believe that a Bayesian approach, which averages over the uncertainty of the model parameters, provides additional modeling flexibility allowing for more reasonable and reliable inferences. Throughout the article, we apply vague normal priors for our key parameters of interest. So as to simulate the Bayesian posteriors, we employ a Metropolis algorithm empowered within JAGS so as to obtain marginal posterior densities for the key parameters, as well as the alternative-specific constants  $\lambda_j$  which are included to guard against estimation bias and pick up alleged “valence effects”.

In each of the ninety national election contexts, we specify four Marlow chains with diverse initial values and save 100,000 iterations after a 50,000 iteration burn-in. Note that our ninety parallel conditional logit models pass all conventional model diagnostics such as the Geweke and Heidelberger-Welch tests of non-stationarity. Thinning, i.e. only retaining every  $n$ -th simulation interval, enables us to deal with substantive auto-correlation within and comparatively slow mixing among the four Markov chains. The actual MCMC simulation gains additional efficiency by “blocking” the key model parameters  $\alpha$  and  $\beta$  (for theoretical and practical issues of Bayesian model estimation cf. Gelman et al., 2013; Jackman, 2009).

Thus, our comparative analyses involve a whole battery of estimators for the key model parameters, their marginal effects and implied choice probabilities. We have therefore decided to move the summaries of our posteriors to the supplementary material and to instead focus on graphical presentations of our key findings: Fig. 1 displays summaries of our posterior distributions and focuses on parameters for issue salience ( $\hat{\alpha}$ ; x-axis) and attitudes towards risk ( $\hat{\beta}$ ; y-axis). Both panels essentially provide the same information: the upper panel plots coefficient estimates for  $\alpha$  and  $\beta$  and indicates their confidence intervals; the lower panel replaces dots and lines with proper election labels so that (many/most of) the national election contexts may be identified. (For detailed summaries of the individual effect parameters and their posterior distributions please refer to the supplementary material.)

The horizontal dimension of Fig. 1 unambiguously underscores the significance of spatial utilities for vote choice across the set of ninety national election studies covered in the stacked CSES dataset. In all of these very heterogeneous election contexts, spatial proximity among



**Fig. 1.** Posterior Means and Distributions of Saliency and Shape Parameters ( $\hat{\alpha}$  and  $\hat{\beta}$ ). **Notes:** These plots summarize posterior densities of the saliency and shape parameters in our baseline models of voting under uncertainty. Empirical results are based on ninety country-specific conditional logit models. General electoral utility is provided by  $u(v, p) = -\alpha|v_i - p_{i,j}|^\beta + \theta_j D_i + \lambda_j c_j$ .  $\alpha$  indicates the saliency of ideological proximity among voters and party alternatives, and  $\beta$  indicates the risk or shape parameter for each national election context. Note that the individual elections are labeled by the three-digit ISO country code and the election year. Detailed results are part of the replication material on the journal website.

voters and parties is closely and substantively meaningfully linked with reported vote choice (aka  $\hat{\alpha} > 0$  for all election contexts). The coefficients indicate that the spatial model of voting provides valuable insights into electoral choice and holds robustly across diverse social, political, and institutional settings. Both in abstract formal theory and in empirical and statistical models of vote choice, voters prefer parties which more closely match their personal ideal points to others which are located further away within our uni-dimensional political space.

While spatial proximity on the left-right scale thus impacts on vote choice throughout the diverse country and election contexts in the data at hand, the magnitude of the effect and the salience of this ideological dimension varies considerably. By and large, ideological proximity among voters and party platforms appears to be less salient in some newer democracies or when party systems remain in a state of flux (cf. some elections in Albania, Brazil, or Peru) and significantly more salient in democracies which are characterized by significant levels of political polarization (such as Denmark, Hungary, or Spain). In this brief article, we can only suggest some tentative explanations for the reasons underlying these differences: some electoral contexts may be more straightforwardly characterized by ideologically driven party competition than others, and, as a result, the uni-dimensional conflict of left and right may be more characteristic for some contexts than others.

The vertical dimension of Fig. 1 displays posterior means and confidence intervals of our key quantity of interest, the shape parameter of voter utility functions. The findings derived from our series of conditional logit models illustrates that the obtained utility profiles gravitate towards risk-neutral attitudes ( $\hat{\beta} \sim 1$ ). Not in a single election segment, the posterior mean  $\hat{\beta}$  was in the domain of extreme risk aversion or close to the strictly risk-averse standard as assumed by quadratic utility functions. In a number of countries and elections, for instance in Japan, South Africa, and the Ukraine, voters appear to be vaguely risk-averse, while in others, such as Bulgaria, Portugal, or Taiwan, average utility profiles are at least partly convex and a majority of voters appears to be ready to take electoral gambles.

These substantive results on the shape of utility functions challenge some core assumptions in the formal and empirical literature: Average utility curves are regularly convex instead of concave, voters are generally not risk-averse, but the electorates in the majority of the heterogeneous election contexts hover towards risk-neutral assessments and behavior. The strong assumption of strictly concave utility profiles which lies at the heart of (most) standard spatial voting models is far too restrictive and not backed up by the rich empirical data at hand. These results reinforce the previous findings in the literature which are, however, based on limited empirical data taken from single elections (cf. Grynawski and Corrigan, 2006; Thurner, 2000) or more restrictive theoretical and statistical models (cf. Singh, 2014).

Concluding this part of the article, we also need to attach a word of caution to our interpretation of these results. A key concern is that our findings appear to be to a certain extent model dependent. Regardless whether one identifies the shape parameter  $\beta$  by some estimation or simulation procedure or whether one fixes its value à priori by either selecting linear (setting  $\beta = 1$ ) or squared distances (setting  $\beta = 2$ ), the numerical magnitude of the salience parameter  $\alpha$  depends on the choice of or the estimate for the shape parameter  $\beta$  (and vice versa). Focusing on the spatial component of the formal utility specification, it is very obvious that the salience and shape parameters are inversely related and one coefficient estimate increases at the expense of the other (roughly  $-\alpha \sim \beta$ ). Turning towards the empirical evidence, Fig. 1 clearly reveals that, across our ninety election segments, the level of  $\hat{\alpha}$  partly depends upon the level of  $\hat{\beta}$  and thus cannot be meaningfully interpreted in isolation.

#### 4.2. Comprehensive models of vote choice under uncertainty

Evidently, any assessment of attitudes towards risk also needs to

consider voter uncertainty to arrive at realistic and substantively meaningful. We complete our statistical model through the systematic consideration of voter uncertainty about party locations within the political space. The measurement of uncertainty is supposed to be founded upon standard survey material as included, for instance, in the CSES questionnaires. However, measuring uncertainty is often an uncertain endeavor per se and comes with many conceptual issues, problems, and with sometimes contestable conceptual decisions (for a detailed review cf. Alvarez, 1998).

Focusing on individual-level measures, two principal approaches have emerged in the literature: Bartels (1986) argues that all voters are uncertain about all party locations to a certain extent. However, when the level of uncertainty exceeds a certain threshold, survey respondents will no longer be able to locate a party and thus refuse to respond to the related survey items. While uncertainty hence may not be observed directly, patterns of issue non-response are regarded as indications of voter uncertainty, and in turn uncertainty is systematically related to observable features of the survey respondents and the parties. Subsequently, Bartels (1986) moves on to modeling and predicting issue non-response by variables such as a voters gender, race, age, party identification, levels of political information, interest in politics, or exposure to campaign messages.

Instead of relying on inferential strategies, Alvarez (1998, 53–75) insists on the preference of directly observing uncertainty indicators. His measurement strategy assumes that any individual-specific party placement  $p_{i,j}$  consist of a “true” party placement  $p_j$  and a voter-specific term indicating perceptual error or uncertainty  $v_{i,j}$ :  $p_{i,j} = p_j + v_{i,j}$ . Voter uncertainty is therefore provided by  $v_{i,j}$ , and is assessed by the mismatch of individual-specific and “true” placements so that  $v_{i,j} = |p_{i,j} - p_j|$ . If, however, a respondent does not indicate any position for a specific party on a specific issue dimension,  $v_{i,j}$  is assigned the maximum distance on that dimension, i.e. with eleven-point scales, she is assigned the maximum scale difference of  $v_{i,j} = 11$ .

We now connect measurement with modeling issues and describe the representation of voter uncertainty in our statistical models of vote choice. Previous contributions which assume quadratic, strictly risk averse loss functions, have exploited the mean-variance composition (as described in Appendix Appendix A) and concluded that uncertainty about party platforms always impedes voter utility (Alvarez, 1998; Bartels, 1986; Shepsle, 1972). In this article, we instead follow and revise an approach that has been pioneered by Berinsky and Lewis (2007) and replicated by Shikano and Behnke (2009) for selected elections to the American presidency and the German “Bundestag”. We continue to model vote choice by standard conditional logit models. However, instead of pasting a crisp, fixed placement for each party competing, we assign a probability distribution to the alternative-specific party placements and, in each iteration of the Bayesian model, draw an another value from this distribution. This modeling strategy exactly reflects our idea that uncertainty about the ideological or political position of a party alternative may be appropriately captured by assigning a probability distribution over likely positions instead of pasting a fixed and unified placement that is actually unknown to the voters.

We connect these basic measurement and modeling approaches, i.e. direct assessments of uncertainty ( $v_{i,j}$ ) and the assignment of probability distributions, to party positions, by the assumption that both are proportional. The more uncertain a voter is about the placement of a specific party  $p_{i,j}$ , the higher the variance of the probability distribution over its position:  $\sigma^2(p_{i,j}) \propto v_{i,j}$ . This modeling strategy implies that instead of pasting a crisp, unified party position  $p_{i,j}$  into the statistical model, we capture uncertainty about the political positions of alternative parties by assigning a (normal) distribution over the perceived party locations. Following Bartels (1986, 717), our measure of uncertainty captures variances of perceived party positions proportional to an unknown scale factor. Therefore, the statistical models need to rescale our probability distributions by an additional coefficient  $\gamma$

which indicates the impact of uncertainty relative to spatial distances. Substantively, the expression  $\gamma\sigma [p_{i,j}]$  may be evaluated as the actual, empirical variation assigned to each party position perception by each voter. Formally, voter uncertainty is therefore represented by a normal distribution centered over individual party placements:

$$\sigma^2(p_{i,j}) \propto v_{i,j}$$

$$\sigma^2(p_{i,j}) = \gamma v_{i,j}; \gamma \sim U(0,10)$$

In the course of the Bayesian model simulation, we need to ensure that standard deviations or variances are always positive. The first component required for computing the variance, the empirical uncertainty parameter  $v_{i,j}$ , is positive by definition. To ensure that the second product term, the rescaling parameter, is also always positive, we parameterize  $\gamma$  by a weakly informative uniform prior distribution  $U[0,10]$  ranging from zero to ten (cf. Jackman, 2009; Gelman and Hill, 2006).

Subsequently, we utilize these variance terms to construct to construct voter- and party-specific uncertainty distributions  $\Theta_{i,j}$  and, instead of crisp party placements, paste (draws from) these distributions into the expressions of voter utility and, in turn, the random utility model:

$$\Theta_{i,j} = N(p_{i,j}, \sigma^2[p_{i,j}]) = N(p_{i,j}, \gamma v_{i,j})$$

$$u(v, \Theta_{i,j}|v_{i,j}) = -\alpha|v_i - \Theta_{i,j}|^\beta + \lambda_j c_j$$

$$\Pr(v_i = j|v_{i,j}) = \frac{\exp[u(v, \Theta_{i,j})]}{\sum_{j=1}^J \exp[u(v, \Theta_{i,j})]}$$

We specify a series of election-specific conditional logit models, and the parameter estimation is performed by MCMC; we use 100,000 iterations after a 50,000 iteration burn-in. Fig. 2 presents summaries of the posterior distributions for the salience parameter  $\hat{\alpha}$ , the shape

parameter  $\hat{\beta}$ , and the uncertainty parameter  $\hat{\gamma}$ . (Details of the model implementation in JAGS are documented in Appendix C.) The left-hand panel (a) shows posterior distributions for the salience parameter. With uncertainty incorporated into the statistical models, our MCMC estimates for the salience of the left-right dimension are confined to by and large identical ranges. Across the board, spatial proximity on the left-right scale continues to exert a substantive impact on vote choice, the series of posterior means  $\hat{\alpha}$  is persistently positive, and the related confidence regions of the posterior distribution are always larger than zero. Furthermore, it is important to note that the consideration of voter uncertainty does not significantly change the findings presented above. Posterior means of  $\hat{\alpha}$  obtained from voting under certainty and uncertainty correlate very highly.

Switching from issue salience towards risk attitudes of the electorates, the center panel (b) displays posterior distributions of the shape or risk parameters. Even with uncertainty about party locations explicitly incorporated into the statistical models, the individual survey segments continue to gravitate towards risk-neutral rather than risk-averse attitudes of the electorates. The posterior means of  $\hat{\beta}$  refer to risk-averse electorates in some survey segments (shown at the top of the center figure), but the majority of the electorates does not tend to be systematically different from roughly risk-neutral attitudes ( $\beta \sim 1$ ). None of the obtained shape parameters  $\hat{\beta}$  comes close to the Neo-Downsian standard model of  $\beta = 2$  and strictly concave, quadratic loss functions. As before, the incorporation of voter uncertainty does not systematically reshuffle the obtained shape parameters, and posterior means  $\hat{\beta}$  obtained from models with and without uncertainty correlate very significantly.

Finally, the right-hand panel (c) illustrates the effects of voter uncertainty on electoral choice. We have already underscored that average values of the directly observed uncertainty indicator  $v_{i,j}$  differ significantly from one country or election to another. The uncertainty

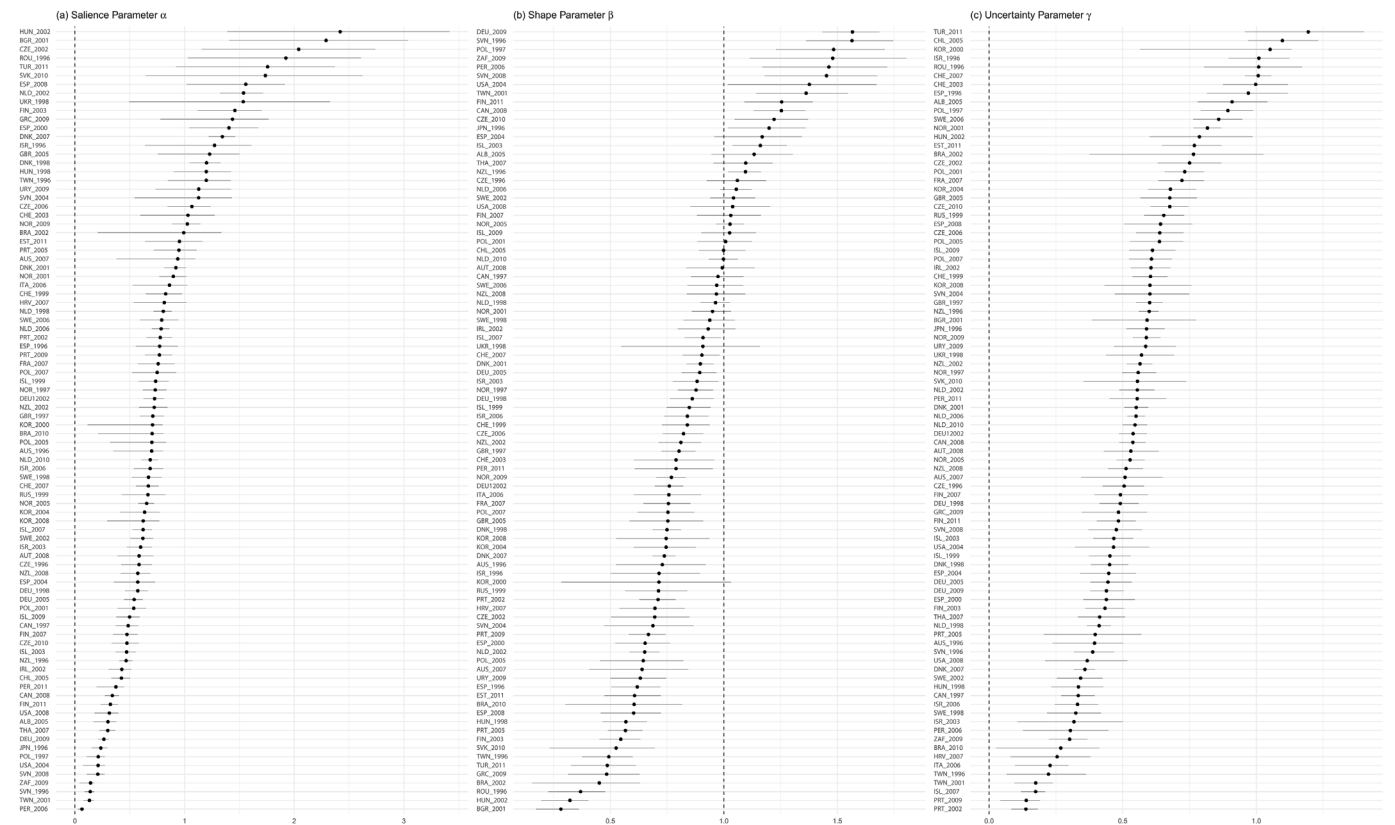


Fig. 2. Posterior Means and Distributions of Salience, Risk and Uncertainty Parameters ( $\hat{\alpha}$ ,  $\hat{\beta}$ , and  $\hat{\gamma}$ ). Notes: General electoral utility is provided by  $u(v, p|v_{i,j}) = -\alpha|v_i - \Theta_{i,j}|^\beta + \lambda_j c_j$ . The coefficient plots summarize posterior densities of the salience ( $\hat{\alpha}$ ), risk ( $\hat{\beta}$ ), and uncertainty parameters ( $\hat{\gamma}$ ). For  $\hat{\alpha}$  and  $\hat{\gamma}$ , the vertical dashed lines indicate no effects of issue proximity or voter uncertainty; for  $\hat{\beta}$  the dashed line indicates risk-neutral voters.

parameter  $\hat{\gamma}$  governs how voter uncertainty is rescaled and thus provides a standard distribution over uncertain positions of each party  $p_{i,j}$ . Larger values of  $\hat{\gamma}$  imply that voter uncertainty impacts very prominently on electoral behavior, smaller values ( $\hat{\gamma} \sim 0$ ) indicate that voter uncertainty is (almost) inconsequential for empirical explanations of vote choice. While our empirical estimates underscore that voter uncertainty does exert a significant impact on vote choice, the individual- and context-specific impacts vary considerably.

Each party placement by each voter is characterized by a normal distribution with a variance of  $\sigma v_{i,j}$ , i.e. the rescaling coefficient  $\gamma$  times the observed uncertainty parameter  $v_{i,j}$ . The political consequences of voter uncertainty differ widely among voters, parties, elections, and countries. Given similar levels of voter uncertainty  $v_{i,j}$ , these distributions are rather wide in Turkey or in Switzerland, while they are narrow in countries like Portugal or Taiwan. Across the ninety election surveys at hand, the mean value of  $\sigma^2 \approx 0.56$ . In a hypothetical “average” country and for a party the respondent could not locate and which was thus assigned the maximum uncertainty score of  $v_{i,j} = 11$ , the variance amounts to  $\sigma^2 = 0.56 \cdot 11 \approx 6.16$ , and the 95 percent confidence region is as wide as  $\pm 4.86$  scale points. If, everything else being equal, the respondent misplaces the party only by two scale points, the variance reduces to  $\sigma^2 \approx 1.12$ , and the confidence region shrinks to  $\pm 2.07$  scale points from the “true” party position.

Given that in the most of the country and election contexts between four and six viable political parties compete for the votes, the voter-, and party-specific uncertainty distributions are likely to overlap very regularly. As a result, the introduction of voter uncertainty impacts on voter utilities, their ordering, and most likely also on vote choice.

#### 4.3. Salience, risk, and uncertainty in context

Eventually, we present some careful, tentative considerations concerning the causal underpinnings of risk attitudes. Searching for systemic features of the obtained salience ( $\hat{\alpha}$ ), risk ( $\hat{\beta}$ ), and uncertainty parameters ( $\hat{\gamma}$ ), we begin by examining whether individual elections taken from the same national context are systematically similar to one another. So as to assess the empirical “stickiness” of our key parameters, we compute intra-class correlations for the posterior means  $\hat{\alpha}$ ,  $\hat{\beta}$ , and  $\hat{\gamma}$ . In the stacked CSES dataset, the number of elections per polity differs considerably. For some countries, there is only one election over the entire time span from 1996 to 2011, while we have data on up to four election surveys nested in other country contexts.

Fig. 3 presents intra-class correlations of generic coefficients obtained from both baseline models of vote choice with certain party placements and comprehensive models under uncertainty. Across the board, and both for models specified under certainty and under uncertainty, the intra-class correlation coefficients  $\rho$  refer to substantial levels of intra-class correlation among  $\hat{\alpha}$ ,  $\hat{\beta}$ , and  $\hat{\gamma}$  within stable country contexts. The empirical levels of autocorrelation are consistently high among the coefficients which pick up the general salience of the ideological divide ( $\hat{\alpha}$ ) and the electoral consequences of party identification in either model. But autocorrelation is also significant substantively meaningful for the coefficients on risk ( $\hat{\beta}$ ) and uncertainty ( $\hat{\gamma}$ ).

Systematic empirical autocorrelation of key coefficients and their respective impacts within the national contexts clearly refers to the relevance of stable political institutions such as party systems, stable patterns of political competition, and institutional architectures. In our empirical analysis, however, we have not found any systematic associations the shape and uncertainty parameters with contextual predictors such as basic features of electoral systems, levels of electoral or parliamentary fragmentation, or party polarization. Furthermore, we could not establish any empirical links of  $\hat{\alpha}$ ,  $\hat{\beta}$ , and  $\hat{\gamma}$  with indicators that capture party and party system institutionalization or the programmatic crystallization of political party systems. Clearly, the whole

field of contextual impacts on key spatial voting parameters warrants additional attention and systematic research.

## 5. Conclusion

The empirical findings presented above are significant from a number of different angles. In conceptual and theoretical terms, voter attitudes towards risk are of direct relevance for the correct and realistic model or specification of vote choice. While previous approaches turn towards squared spatial distances as a matter of routine and mathematical convenience, the empirical findings in this contribution explicitly contradict any strong notion of globally risk-averse voters. Theoretical and statistical models which à priori assume quadratic Euclidean distances and both strictly and strongly risk-averse voters are most likely misspecified. Whenever there is evidence that voters (and perhaps other agents) have at least partly convex preference curves, standard items in the literature such as the instability of majority rule, would likely be under pressure (Plott, 1967; McKelvey and Wendell, 1976; Schofield, 1978).

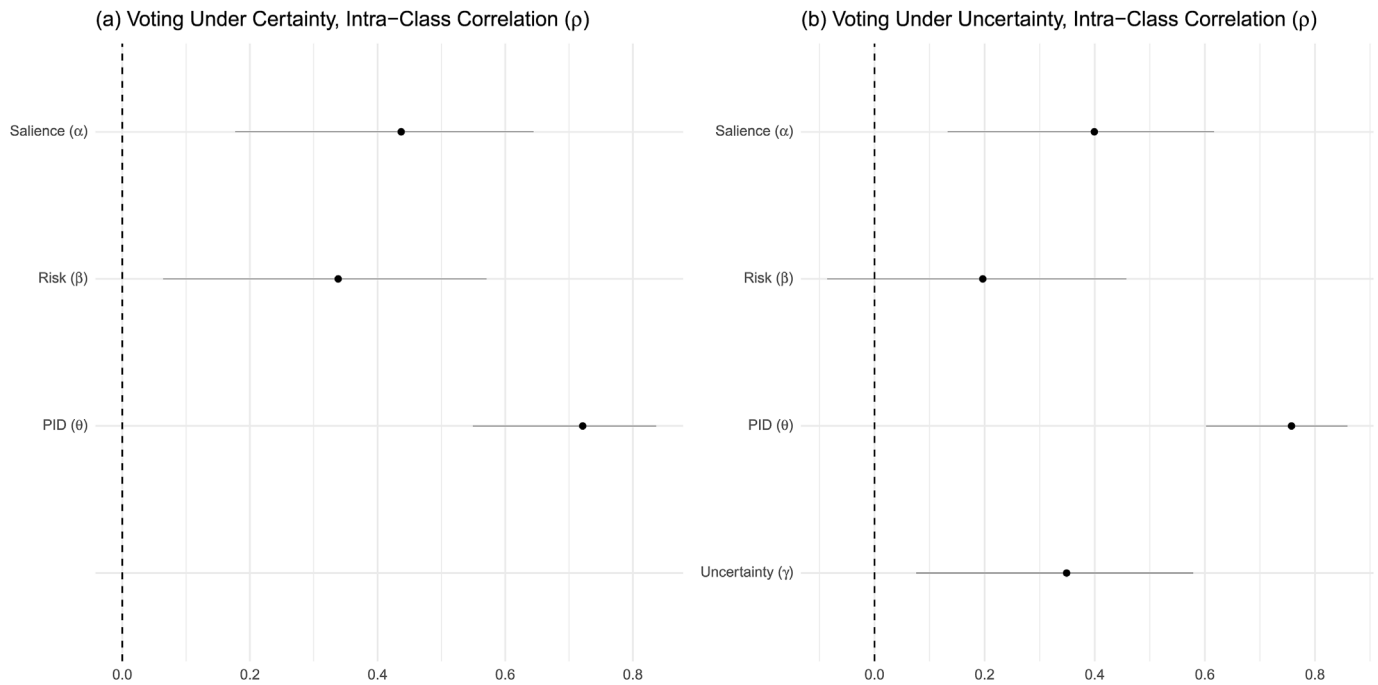
Nevertheless, there are still many plausible reasons to prefer quadratic loss function or to continue to apply them as an approximation of the “true” loss function even when they are unrealistic and not supported by empirical findings. Particularly, Euclidean utility loss functions are mathematically convenient, since quadratic loss functions are differentiable at the ideal point and allow for simple mean-variance decomposition in models involving voter uncertainty (Alvarez, 1998; Bartels, 1986; Enelow and Hinich, 1981; Shepsle, 1972).

Moving towards more applied and substantive matters, if these empirical findings can be sustained and voters are much less risk-averse than previously assumed (and modeled), the dynamics of vote choice and party competition are also likely affected. Voters who are indifferent towards risk do not systematically punish candidates or parties that lay out blurred and unclear ideological or programmatic platforms. Therefore, a campaign plan of offering “Everything to Everyone” (Somer-Topcu, 2015) may actually be the winning strategy for modern elections. Tactical party elites may also lay out more complex blends of certain and uncertain policy issues in order to emphasize/deemphasize alternative dimensions of political contestation. Perhaps even more significantly, dissatisfied voters who have lost trust in democratic procedures and the impact, and thus the value, of their vote may be encouraged to accepted odd-sized electoral gambles (cf. the experimental findings by Tomz and Van Houweling, 2009).

While this article addresses a number of key theoretical and substantive concerns, there are still important aspects that warrant further research: Most importantly, our analysis can only focus on the risk attitudes of entire electorates and thus needs to assume that these orientations are stable across diverse voters and party alternatives. Given that this does not appear to be a realistic scenario, more needs to be done to systematically include items that capture risk attitudes and proxies of party ambiguity and voter uncertainty in election survey. Even more preferably, causal inferences could be sustained by broadening and continuing the experimental approach pioneered by Tomz and Van Houweling (2009) to also consider multiparty competition and apply to party ambiguity and voter uncertainty.

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**Fig. 3.** Intra-Class Correlations of Saliency ( $\hat{\alpha}$ ), Risk ( $\hat{\beta}$ ), and Uncertainty Parameters ( $\hat{\gamma}$ ). . **Notes:** Throughout the article, we compute separate, election- and survey-specific discrete choice models for ninety election surveys which are nested in forty-four polities. These plots display intra-class correlations and the associated confidence intervals. Details on the computation of these coefficients are provided by Li and Baron (2012, 220–223).

### Appendix A. Risk-Averse Voters and Perceptual Uncertainty

The theoretical literature has predominantly applied formal models to explore the effects of party ambiguity and voter uncertainty on vote choice and the electoral performance of political parties. As a common point of departure, a series of contributions assumes voters to be consistently risk-averse and employs quadratic loss functions to capture spatial utility (Alvarez, 1998; Bartels, 1986; Enelow and Hinich, 1981, 1984; Shepsle, 1972).

In this strand of the literature, the individual contributions have applied very similar formal proofs, which all build upon the expression of idiosyncratic party placements  $p_{i,j}$  as the sum of a “true” party position upon a specific dimension  $p_j$  and an alternative-, individual-, and issue-specific concept of perceptual uncertainty  $v_{i,j}$ . Next, this expression is pasted into a conventional spatial voting model which assumes risk-averse voters and thus implies strictly quadratic spatial loss functions. After rearranging terms, the addition of perceptual uncertainty into a spatial voting model with idiosyncratic party placements yields a revised utility function. Most importantly, the utility of a party to a voter now depends upon a quadratic loss function of voter ideal points and “true” party positions, while idiosyncratic perceptions merely enter electoral utility via the uncertainty penalty in the model.

The subsequent argument closely follows the argument formalized by Alvarez (1998, 30–36) which, in turn, follows previous formal demonstrations by Enelow and Hinich (1981), Enelow and Hinich (1984), and Bartels (1986). Idiosyncratic, individual-specific party placements on any issue dimension ( $p_{i,j}$ ) may conceptually be decomposed into the “true”, unified party position  $p_j$  and the variance of voter perceptions  $v_{i,j}$ :  $p_{i,j} = p_j + v_{i,j}$ . The political consequences of voter uncertainty with risk-averse voters may be demonstrated by plugging this expression into the conventional quadratic loss function:

$$\begin{aligned}
 U_{i,j} &= \sum [- (v_i - p_{i,j})^2] + c_j \\
 E[U_{i,j}] &= E[\sum [- (v_i - p_{i,j})^2] + c_j] \\
 &= E\left[\sum \left[-\left(v_i^2 - 2v_i p_{i,j} + p_{i,j}^2\right)\right] + c_j\right]
 \end{aligned}$$

Next, we move the expectations operator through the right-hand side of the equation and note that  $E[c_j] = c_j$ ;  $E[v_i] = v_i$ ;  $E\left[p_{i,j}^2\right] = p_j^2 + v_{i,j}^2$ ;  $E[v_i v_{i,j}] = 0$ ;  $E[v_{i,j}^2] = \sigma_{i,j}^2$ , and  $E[p_{i,j}] = p_j$ . Collecting terms on the right-hand side of the equation yields:

$$E[U_{i,j}] = \sum [- (v_i - p_j)^2 - \sigma_{i,j}^2] + c_j$$

Formal models from this tradition regularly postulate that party ambiguity and other effects that contribute to voter uncertainty depress electoral utility and reduce the electoral chances of ambiguous and/or unclear political actors vis-à-vis their (more) precise competitors.

### Appendix B. Data and Variables

#### Appendix B.1. Survey Segments in the CSES Trend File

The empirical analyses presented in this article are based upon the rich material provided by “The Comparative Study of Electoral Systems”

(CSES). More specifically, we utilize the “CSES Harmonized Trend File” which gathers and standardizes country- and election-specific survey segments from the first three waves of the CSES project, i.e. from 1996 to 2001. The integrated dataset has been compiled at the “Berlin Social Science Center” by Heiko Giebler, Josephine Lichteblau, Antonia May, Reinhold Melcher, Aiko Wagner and Bernhard Weßels. Data and documentation are available via the CSES project site (<http://cses.org/datacenter/trendfile/trendfile.htm>).

The CSES trend file includes variables which have been included in repeated survey waves. Altogether, the CSES trend file unifies parallel survey modules from 129 individual survey segments which have been gathered in the contexts of parliamentary and presidential elections. So as to ensure the comparability of electoral contexts, we have only included elections to national parliaments (to the lower houses in bicameral systems), but we have excluded more personalized presidential elections. Moreover, we could not consider a number of other elections: In some cases, key variables of the spatial model or key control variables are missing or the data has not been consistent; some survey segments unfortunately had a very small N which prevented meaningful inferences. Building on these criteria, we maintained ninety of the originally 129 survey segments for the comparative analysis:

Albania (2005), Austria (2008), Australia (1996, 2007), Bulgaria (2001), Brazil (2002, 2010), Canada (1997, 2007), Chile (2005), Croatia (2007), the Czech Republic (1996, 2002, 2006, 2010), Denmark (1998, 2001, 2008), Estonia (2011), Finland (2003, 2007, 2011), France (2007), Germany (1998, 2002, 2005, 2009), Great Britain (1997, 2005), Greece (2009), Hungary (1998, 2002), Ireland (2002), Iceland (1999, 2003, 2007, 2009), Israel (2003, 2006), Italy (2006), Japan (1996), the Netherlands (1998, 2002, 2006, 2010), Norway (1997, 2001, 2005, 2009), New Zealand (1996, 2002, 2008), Peru (2006, 2011), Poland (1997, 2001, 2005, 2007), Portugal (2002, 2005), Romania (1996), Russia (1999), Switzerland (1999, 2003, 2007), Slovakia (2010), Slovenia (1996, 2004, 2008), Sweden (1998, 2002), Spain (1996, 2000, 2004, 2008), South Africa (2009), South Korea (2000, 2004, 2008), Thailand (2007), Turkey (2011), Taiwan (1996, 2001), Ukraine (1998), Uruguay (2009) and the United States (2004, 2008).

## Appendix B.2. Selected Variables in the CSES Trend File

This appendix presents some brief information on the key and control variables which we have used in the empirical analyses. The brief descriptions below refer to the variable names in the “CSES Module 1–3 Harmonized Trend File” (<http://cses.org/datacenter/trendfile/trendfile.htm>).

### Appendix B.2.1. The Key Spatial Model

First, we include some key variables which capture key data on the spatial model of voting:

*iA3031\_m*: left-right self-placements are evaluated on an eleven-point scale ranging from zero to 10; note that survey respondents who did not locate themselves on the ideological scale have been excluded from the analysis;

*iA3032\_m*: alternative-specific left-right placements are evaluated on eleven-point scales ranging from zero to ten for up to nine electoral platforms;

### Appendix B.2.2. Controls

Secondly, we also included a number of control variables:

*PID\_closest*: closeness to and identification with a political party; survey respondents are inquired to indicate a party they feel closest to, and this information is broken down to an alternative-specific binary variable which indicates whether a person identifies with a party or not;

WHICH PARTY DO YOU FEEL CLOSEST TO?

*iA3001\_m*: satisfaction with the democratic process;

SATISFACTION WITH DEMOCRATIC PROCESS?

(1) very satisfied; (2) fairly satisfied; (3) not very satisfied;

(4) not at all satisfied

*iA2023\_m*: political information item;

1ST POLITICAL INFORMATION ITEM

(0) incorrect answer, missing, don't know; (1) correct

*iA2024\_m*: political information item;

2ND POLITICAL INFORMATION ITEM

(0) incorrect answer, missing, don't know; (1) correct

*iA2025\_m*: political information item;

3RD POLITICAL INFORMATION ITEM

(0) incorrect answer, missing, don't know; (1) correct

The three political information items cover country-specific political knowledge items and are summarized so as to obtain a political knowledge index ranging from zero (=no correct answer) to three (=three correct answers).

## Appendix C. JAGS Code

This appendix provides the JAGS code which has been utilized to simulate uncertainty over alternative-specific party positions. Note that, due to the “blocking” of key parameters of interest, some parameter names differ somewhat from the usage in the previous theoretical and statistical

models. Specifically,  $\alpha[1]$  labels the salience weights (“ $\alpha$ ” in the main text), and  $\alpha[2]$  denotes the form parameters (“ $\beta$ ” in the main text). The matrix of alternative-specific constants is denoted by  $\beta[i,j]$ .

```

model{
  for(i in 1:N_V){
    for(j in 1:N_P){
      # utility function;
      mu[i,j] <- beta[i,j] - alpha[1] * (abs(lr_i[i]-lr_ij.dist[i,j])^alpha[2])

      # random-utility model;
      emu[i,j] <- exp(mu[i,j])
      p[i,j] <- emu[i,j]/sum(emu[i,1:N_P])

      # uncertainty distributions;
      prec.lr[i,j] <- pow(unc_ij[i,j] * gamma,-2)
      lr_ij.dist[i,j] ~ dnorm(lr_ij[i,j], prec.lr[i,j])
    }
    vote[i] ~ dcat(p[i,1:N_P])
  }
  # PRIORS ;
  # ALPHA (for alt.-specific predictors);
  alpha[1:N_ALPHA] ~ dnorm(a0, A0)
  # BETA (for ind.-specific predictors);
  # reference category ;
  for(i in 1:N_BETA){
    beta[i,1] <- 0
  }
  for(i in 2:N_P){
    beta[1:N_BETA,i] ~ dnorm(b0,B0)
  }
  # GAMMA ;
  gamma ~ dunif(0,1)
}

```

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